**Barron’s Let’s Review Regents – Algebra I**

# Chapter 5: Graphs of Solution Sets of Linear Equations

## 5.1 Producing a Graph by Identifying Two or More Points

A linear equation, like x + y = 10, is one where neither of the variables has an exponent greater than or equal to 2. The set of ordered pairs that makes this equation true includes (2, 8), (3, 7) and (9, 1). If each ordered pair is plotted as a point on the *coordinate plane*, the result is the *graph of the solution set of the equation*.

### Graphing the Solution Set of Linear Equation Making a Table of Values

The equation x + y = 10 has an infinite number of ordered pairs that satisfy it. For a linear equation, only two ordered pairs are needed.

|  |  |
| --- | --- |
| x | y |
| 2 | 8 |
| 3 | 7 |
| 9 | 1 |

The line on a graph contains an infinite number of points.

### Graphing the Solution Set of Linear Equations Using the Two-Intercept Method

Unless a line is vertical or horizontal, it will cross both the x-axis and y-axis. The point where the line crosses the x-axis is called the *x-intercept*, and the point where it crosses the y-axis is called the *y-intercept*. Any point on the y-axis has an x-coordinate of 0, and any point on the x-axis has a y-coordinate of 0.

A quick way to make a graph of the solution set of a two-variable equation is to calculate the x- and y-intercepts.

### Equations for Horizontal or Vertical Lines

An equation with one variable can also have a solution set of ordered pairs. One ordered pair that satisfies the equation y = 3 is (0, 3). When all ordered pairs that satisfy the equation y =3 are graphed, it becomes a horizontal line with y-intercept of (0, 3).

### Graphing Linear Equations Involving Absolute Value

The absolute value of a number is defined as the distance that number is from zero on the number line. The symbol for the absolute value of a is |a|.

### Solving Systems of Linear Equations by Graphing

The ordered pair that is the solution to a system of linear equations will be related to the point of intersection of the two lines that are graphs of the solution sets of the two equations.

x + y = 10  
x – y = 2

Using algebra, the solution was (6, 4).

A graph of x and y with a green line

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### Check Your Understanding of Section 5.1

1. Multiple-Choice
2. Which are the coordinates of a point that will be on the line that represents the solution set of the equation 2x + 3y = 12?  
   **(4) (3,2)**
3. What is the x-intercept of the graph of the solution set of the equation 2x + 5y = 20?  
   **(1) (10, 0)**
4. What is the y-intercept of the graph of the solution of the equation 3x – 8y = 24?  
   **(1) (0, -3)**
5. The point (2, k) is on the graph of the solution set of the equation 3x + y = 15. What is the value of k?  
   **(3) 9**
6. Based on the x-intercept and y-intercept, this is the graph of the solution of which equation?  
   x-intercept (9, 0), y-intercept (0, 2)  
   **(2) 2x + 9y = 18**
7. Which is a graph of the solution set of the equation y = 5?  
   **(2)**
8. This is a graph of the solution set of which equation?  
   **(1) x = 8**
9. The equations 2x – 3y = 9 and 3x + 2y = 20 are graphed below. What is the solution to the set of equations?  
   A graph of x and y with red line

   AI-generated content may be incorrect.  
   **(4) 6, 1**
10. Below is the graph of the solution set of an equation. Based on this graph, which ordered pair does not seem to be part of the solution set of the equation ?  
    **(3) (6, 8)**
11. What is the equation of the x-axis?  
    **(2) y = 0**
12. Show how you arrived at your answers.
13. Make a table of values to graph the solution set of the equation y + 2x = 8.

|  |  |
| --- | --- |
| x | y |
| 0 | 8 |
| 4 | 0 |

1. Determine the x-intercept and y-intercept for the graph of the solution set of 4x – 6y = 24 and use them to produce a sketch of the graph.

|  |  |
| --- | --- |
| x | y |
| 0 | -4 |
| 6 | 0 |

The x-intercept is (6, 0) and the y-intercept is (0,4).

A graph of a line

AI-generated content may be incorrect.

1. Tenley says that this is the graph of the equation y = 3. Ingrid says that this is the graph of the equation x = 3. Who is correct? Explain your reasoning.  
     
   Tenley is correct, because y is always 3, no matter what value of x. Ingrid is incorrect because (0, 3) is a point in the solution set, and that is for x = 0, y = 3, which is not x = 3.
2. Graphically solve the system of equations.  
   x – y = 4  
   3x + 5y = 20  
   5x – 5y = 20  
   8x = 40, x = 5, y = 1

|  |  |  |  |
| --- | --- | --- | --- |
| x – y = 4 | | 3x + 5y = 20 | |
| x | y | x | y |
| 0 | -4 | 0 | 4 |
| 4 | 0 | *≈* | 0 |

A graph of x and y with a green line

AI-generated content may be incorrect.

1. Below is the graph for 2x + y = k with intercepts at (0, 8) and (4, 0). What must the value of k be?  
   2(0) + 8 = k, **k = 8**  
   ck: 2x + y = 8  
   2(0) + 8 = 8 ck  
   2(4) + 0 = 8 ck.

## 5.2 Calculating and Interpreting Slope

The *slope* of a line is a number that measures how steep it is. A horizontal line has a slope of 0. A line with a positive slope goes up as it goes to the right. A line with a negative slop goes down as it goes to the right. The variable used for slope is the letter *m*.

**Calculating the Slope of a Line**

The slope of a line is the amount that it moves up (or down) for every one unit it move to the right.

When two lines have the same slope, they are parallel. If the lines have different slopes, they are not parallel. A special case of non-parallel lines is lines whose slopes have a product of -1. When this happens, the lines are perpendicular. When two fractions have a product of -1, one is the *negative reciprocal* of the other.

A horizontal line has a slope of 0, whereas the slope of a vertical line is said to be *undefined*. Division by 0 is undefined, and the vertical line slope is undefined too.

**Interpreting Slope in a Distance-Time Graph**

In a distance-time graph, the slope of a line segment corresponds to the speed of the thing that is moving.

Distance-Time Graphs for Things That Are Changing Speed

When a car is traveling at a constant speed, the distance-time graph will be a line with the same slope at every interval. If the car changes speed, the slope of the distance-time graph will also change.

### Check Your Understanding of Section 5.2

1. Multiple-Choice
2. Which line has a positive slope?  
   **(2)**
3. Which line’s slope is the greatest number?  
   **(4)**
4. Which is closest to the slope of this line?  
   **(1)**
5. What is the slope of the line that passes through   
   (-2, 1) and (8, 5)  
   **(1)**
6. What is the slope of the line that passes through (2, 8) and (5, -1)?  
   **(3) -3**
7. A line with a slope of passes through (2,1) and the point (10, a). What must the value of a be?  
   Multiply both sides by 8.  
   Add 8 to both side:  
   a = 6 + 1 = 7  
   **(1) a = 7**
8. Line 1 passes through the points (-3, -2) and (3, 8). Line 2 is parallel to line 1. What is the slope of line 2.  
     
   **(2)**
9. Below is a distance-time graph for a bicycle trip. During which time is the cyclist going the fastest?  
   (1) 0 to 2 hours, 15 miles per hour  
   (2) 2 to 3 hours, 10 miles per hour  
   **(3) 3 to 5 hours, 20 mile per hour**(4) 5 to 8 hours, 18 miles per hour
10. A line with a slope of 3 passes through the point (4, 1). Which point will this line not pass through?  
    y = mx + b  
    m = 3  
    1 = 3(4) + b  
    b = 1 – 12 = -11  
    y = 3x – 11  
    (1) (5, 4): 4 = 3(5) – 11 = 15 – 11 = 4  
    (2) (6, 7): 7 = 3(6) – 11 = 18 – 11 = 7  
    (3) (7, 10): 10 = 3(7) – 11 = 21 – 11 = 10  
    (4) (7, 11): 11 ≟ 3(7) – 11 = 21 – 11 = 10 11  
    **(4) (7, 11) I disagree with books answer.**
11. What is the slope of the hypotenuse of this right triangle?  
    m =   
    **(3)**
12. Show how you arrived at your answers.
13. Line 1 passes through the points (-3, 7) and (7, 1). Line 2 is perpendicular to line 1. What is the product of slopes line 1 and line 2?  
      
    “A special case of non-parallel lines is lines whose slopes have a product of -1. When this happens, the lines are perpendicular.”  
      
    **The product of slopes line 1 and line 2 is -1.**
14. A triangle has vertex A at (0,0), vertex B at (2,5), and vertex C at (4, 5). Which side of the triangle has the greatest slope?
15. The slope of a line is , and it passes through the point (-5, 2). What are three other points on this line that have coordinates that are integers?  
    y = mx + b  
    x = 1  
    x = 4  
    x = 4, y = 8 => (4, 8)  
    x = 7  
    x = 7, y = 10 => (7, 10)  
    **(1, 6), (4, 8), (7, 10)**
16. A line passes through the points (-7, 2) and   
    (-3, -1). It also passes through the point   
    (a, -7). What is the value of a?  
      
    point 2: (a, -7), point 1: (-7, 2)  
    Multiply by 4(a - 2)  
    -3(a-2) = -9  
    -3a + 6 = -9  
    -3a = -15  
    **a = 5**
17. Find the x-intercept and y-intercept of the solution set of 4x – 6y = 12, use them to find the slope of the line defined by that equation.

|  |  |
| --- | --- |
| x | y |
| 0 | -2 |
| 3 | 0 |

x-intercept: (3, 0)  
y-intercept: (0, -2)  
Point 2: (0, -2), Point 1: (3, 0)

## 5.3 Slope Intercept Form

When a two-variable equation, like 2x + 3y = 12, is written in this form with the variables on one side of the equation and the constant on the other side of the equation, it is called *standard form*.

When it is written in the form y = mx + b, like   
, where m and be could be fractions, it is called *slope-intercept form*.

When an equation is in slope-intercept form, there is a fast way to graph the solution set. Also, when an equation is in slope-intercept form, it is often quicker to solve for x or y when the other variable’s value is known.

**Graphing the Solution Set of a Linear Equation That Is in Slope-Intercept Form**

1. Plot the point (0, b), which is on the y-axis.
2. If the coefficient of the x-term is not already a fraction, turn it into a fraction by putting the coefficient in the numerator of a fraction and a 1 in the denominator.
3. Starting at the y-intercept you already plotted, move right the number in the denominator of the slope. Then, from where you stopped, , move up (down if it is negative) the number in the numerator of the slope.
4. Draw a line through the y-intercept and the new point. Put arrows on both sides of the line to indicate that it continues forever on both sides.

**Finding the Equation in Slope-Intercept Form When the y-intercept and Another Point Are Known**

### Check Your Understanding of Section 5.3

1. Multiple-Choice
2. Which is the graph of y = 2x – 5  
   y-intercept: (0, -5)  
   x-intercept: (2.5, 0)  
   m = 2  
   **(1)**
3. Which is the graph of ?  
   y-intercept: (0, 4)  
   x-intercept: (, 0)  
   **(2)**
4. What is the slope of the line defined by the equation y = -3x + 4?  
   **(2) -3**
5. What is the y-intercept of the line defined by the equation y = 5x + 2?  
   **(2) (0, 2)**
6. In the equation y = 5, what is the 5?  
   **(1) y-intercept**
7. The line defined by the equation is perpendicular to the line defined by which equation?  
   Perpendicular line has a slope where the product of the two slopes is -1.  
   **(4)**
8. This is the grap of the solution set of which equation?  
   Point 2: (3,6)  
   (Point 1): x-intercept: (-15, 0)   
   y-intercept: (0, 5)  
   **(4)**
9. What are two points on the graph for the equation ?  
   y-intercept: (0, 7)  
     
   **(3) (0, 7) and (5, 3)**
10. What are two points on the graph for the equation ?  
    **(3) (0, -4) and (6, 1)**

|  |  |
| --- | --- |
| x | y |
| 0 | -4 |
| 6 | 1 |

1. What is the slope of the graph with the solution set of the equation   
   y = 2 + 3x?  
   **(3) 2**
2. Show how you arrived at your answers.
3. Identify the slope and the y intercept of the graph for the equation y = 3x – 9.  
   **(slope) m = 3  
   y-intercept: (0, -9)**A graph of a line

   AI-generated content may be incorrect.
4. Jaiden says the graph for the equation 2y = 3x -8 has a slope of 3 and a y-intercept of -8. Edwin says that this is not correct and that the slope is and the y-intercept is -4. Which student is correct?  
   **Divide by 2:   
   Edwin is correct. The equation must be in slope-intercept form: y = mx + b, where m is the slope and b is the y-intercept, to properly determine slope and y-intercept.**
5. A portion of the line that passes through (2,4) and (4, 5) is shown below. What is the   
   y-intercept of this line?  
     
   The slope is , which represents the increase in y with respect to the increase in x as the line moves to the right. Moving to the left, the decrease in y is -1 compared to a decrease of -2 for x.  
   **y-intercept: (0, 3)**
6. Solve the system of equations graphically. An algebraic solution will not be accepted.

|  |  |
| --- | --- |
| x | y |
| 0 | 8 |
| 4 | 0 |

|  |  |
| --- | --- |
| x | y |
| 0 | 4 |
| 6 | 0 |

A graph of x and y with lines

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Solution: (3, 2)

1. What do the equations from which these five lines have in common?  
     
   y = mx + b  
     
   y-intercept: (0, 2)

## 5.4 Graphing Solution Sets to Linear Equations with Graphing Calculator

Though the graphing calculator can’t do the real “thinking” for you, it can be a powerful tool to help with graphing solution sets. It is also possible to graphically solve certain equations to find solution sets without needing to use algebra. This book has instructions for the two most popular calculators, the TI-84 Plus and the TI-Nspire.

Graphing Solution Sets for Linear Equations

y = 2x – 5  
  
A graph of a line

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**Making a Table of Values with the Calculator**

y = |2x – 2|  
  
def f(x):

    return abs((2 \* x) - 2)

for x in range(-5, 5):

    print("%3d %3d" % (x, f(x)))  
  
python .\make-table-of-values.py

-5 12

-4 10

-3 8

-2 6

-1 4

0 2

1 0

2 2

3 4

4 6

A graph of a line

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**Systems of Linear Equations with the Intersect Feature**

Find the solution set for the system of equations:

A graph of x and y with a green line

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### Check Your Understanding of Section 5.4

B. Show how you arrived at your answers.

1. Use your graphing calculator to graph the equation y = x – 3.  
     
   A graph of a line drawn on a grid

   AI-generated content may be incorrect.
2. Use your graphing calculator to graph the equation .  
     
   A graph of x and y axis

   AI-generated content may be incorrect.
3. An algebra equation like 2x – 3 = 5 can be solved graphically by intersecting the two graphs y = 2x – 3 and y = 5. Graph these two equations and find the x-coordinate of the intersection to solve the algebra problem   
   2x – 3 = 5.  
     
   A graph of x and y axis

   AI-generated content may be incorrect.  
   Solution: (4, 5)
4. Use your graphing calculator to solve the system of equations.  
     
   Solution: (-5, 6)  
   A graph of x and y lines

   AI-generated content may be incorrect.
5. Use your graphing calculator to solve the system of equations:

A graph of x and y with red and blue lines

AI-generated content may be incorrect.

## 5.5 Determining an Equation for a Given Graph

Just as it is possible to create a graph of a solution set when given a two-variable equation, it is also possible to find the two-variable equation for which a graph is the solution set. Often this equation is then used to answer other questions about the graph.

**Finding the Equation when the Slope and Intercept are known**

In the equation , the *m* represents the slope, and the *b* is the y-intercept.

**Finding the Equation When the y-intercept and Another Point are known**

If the y-intercept is known, then the b-value in the equation is the same number. With one other point, the slope formula can be used with that point and with the y-intercept to calculate the value for *m*.

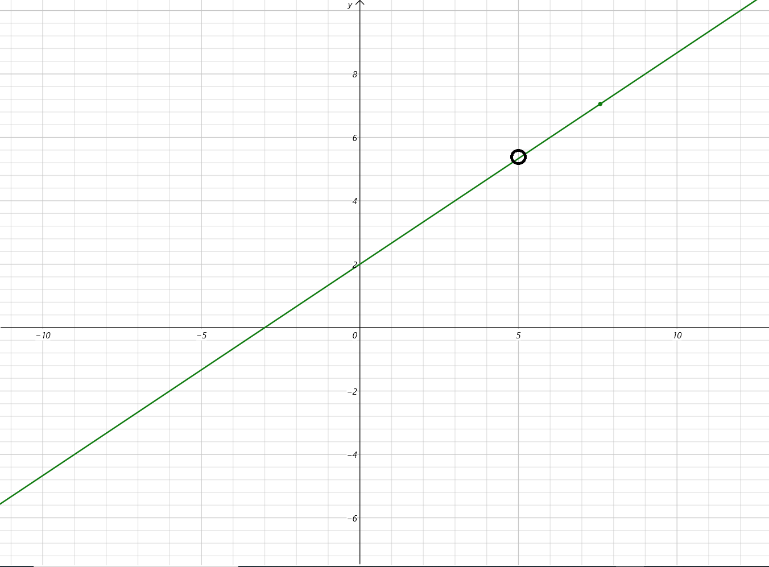
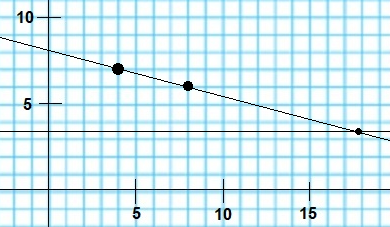
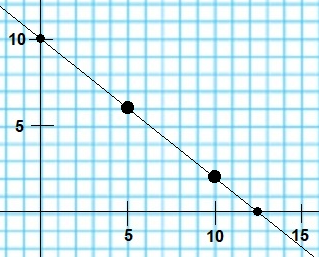
**Finding the Equation When Two Points are Known**

If two points in the solution set for a linear equation are known, neither of which is the y-intercept, there is a two-part process for finding the equation. First, use the two points to calculate m, the slope of the line. The substitute that slope and also the x and y values from one of the points into the equation or to solve for b.

**Using the Calculator to Find the Equation When Two Points Are Known**

If the question is a multiple-choice question about finding the linear equation when two ordered pairs in the solution set are known…

### Check Your Understanding of Section 5.5

1. Multiple-Choice  
   For each question, find the equation of the line that passes through the two given points.
2. (0, -7) and (5, 8)  
   **(2) y = 3x – 7**
3. (0, 5) and (-12, -1)  
   **(3)**
4. (0, 4) and (9, 10)  
   **(3)**
5. (2, 1) and (6, 9)  
   **(1)**
6. (3,1) and (7, -3)  
   **(4)**
7. (4, -2) and (12, 4)  
   **(3)**
8. (5, 1) and (15, -3)  
   **(2)**
9. (-4, 1) and (4, 3)  
   **(4)**
10. (3, 7) and (6, 7)  
    **(2) y = 7**
11. (3, 5) and (3, 8)  
    **(2) x = 3**
12. Show how you arrived at your answers.
13. A line passes through the points (3, 4) and   
    (9, 8). The line also passes through the point (5, a) for what value of a, rounded to the nearest tenth.  
    Point 2: (9, 8)  
    Point 1: (3, 4)  
      
      
    **(5, a), Solution: (5, 5.3)**  
    (5, a)  
    **ck: = 5.3**
14. A line passes through the points (4, 7) and   
    (8, 6). The line also passes through the point (a, 3.5) for what value of a?**Solution: (a, 3.5)**
15. Find the x-intercept and y-intercept of the line that passes through (5, 6) and (10, 2).  
      
      
    **x-intercept: (12.5, 0)  
    y-intercept: (0, 10)**
16. The equation passes through points (6, 7) and (12, a). What is the value of a?  
    **a = 9**
17. The five points (5,4), (6, 5), (7,5), (9, 4) are plotted on a coordinate plane. A line is drawn through the points (5, 4) and (10, 5). This line does not pass through any of the other three points. What is the sum of the three vertical line segments?  
    Point 2: (10, 5)  
    Point 1: (5, 4)

A screenshot of a calculator

AI-generated content may be incorrect.

## 5.6 Word Problems Involving Finding the Equation of a Line

**A Real-World Problem Involving Two Data Points**

### Check Your Understanding of Section 5.6

1. Multiple-Choice
2. A balloon is held so that it is 6 feet above the ground. The balloon is released and 10 seconds later it is 36 feet above the ground. Which equation can be used to relate the time since the balloon was released (T) to the height of the balloon to the height the balloon is above the ground (H)?  
   (T, H),   
   Point 2: (10, 36)  
   Point 1: (0, 6)  
   H = 3T + b  
   36 = 3(10) + b  
   b = 36 – 30 = 6  
   **(4) H = 3T + 6**
3. Lily has $500 in the bank when the year begins. Each week she takes out the same amount of money. After 8 weeks she has $436 left in the bank. Which equation can be used to model this scenario where W is the number of weeks that has passed since the beginning of the year and M is the amount of money remaining?  
   (W, M)  
   Point 2: (8, 436)  
   Point 1: (0, 500)  
   M = -8W + b  
   500 = -8(0) + b  
   b = 500  
   **(3) M = -8W + 500**
4. Marcus puts the same amount of money into the bank each week. Three weeks after the year begins, he has $575 in the bank. Ten weeks after the year begins, he has $750. Which equation can be used to model the amount of money (M) h has in the bank after (W) weeks.  
   Point 2: (10, 750)  
   Point 1: (3, 575)  
   M = 25W + b  
   575 = 25(3) + b  
   b = 575-75 = 500  
   **(2) M = 25W + 500**
5. A taxi costs a fixed amount to get into, and then each mile is an additional fee. If a 4-mile ride costs $8, and a 12-mile ride costs $18, which equation models the Cost (C) of a taxi ride compared to the number of miles (M) traveled?  
   Point 2: (12, 18)  
   Point 1: (4, 8)  
   C = 1.25 M + b  
   18 = 1.25 ( 12) + b  
   18 = 15 + b  
   b = 3  
   **(1) C = 1.25M + 3**
6. Claire exercises each day by running around the track. She increases the amount she runs each day by the same amount. Fifteen days after she begins training, she is running 6 miles. Thirty-five days after she begins training, she is running 10 miles. Which equation relates the number of days she has been training (D) to the number of miles she runs that day (M)?  
   Point 2: (35, 10)  
   Point 1: (15, 6)  
    = 0.2  
   10 = 0.2(35) + b = 7 + b  
   b = 3  
   **(4)**
7. Miriam goes on a diet to lose weight. She loses the same amount of weight each week. If after four weeks she weighs 242 pounds and after seven weeks she weighs 236 pounds, what equation relates her weight in pounds (P) to the number of weeks since she has been on the diet (W)?  
   Point 2: (7, 236)  
   Point 1: (4, 242)  
   P = -2W + b  
   236 = -2(7) + b  
   b = 236 + 14 = 250  
   **(4) P = -2W + 250**
8. When food is put into a special freezer, the temperature of the food decreases by the same number of degrees each minute. If after six minutes, the food is 67 degrees and after twenty minutes the food is 60 degrees, which equation relates the temperature of the food (T) to the number of minutes (M) the food has been in the freezer?  
   Point 2: (20, 60)  
   Point 1: (6, 67)
9. Jocelyn jumps out of an airplane with a parachute. She falls the same number of feet each second. Ten seconds after jumping she is 3,000 feet in the air. Fifteen seconds after jumping she is 2,000 feet in the air. Which equation relates her height in the air (H) to the number of seconds since she jumped (S)?  
   Point 2: (15, 2000)  
   Point 1: (10, 3000)
10. Isaiah and Jaxson are climbing up a mountain. Four hours after starting, they are 4,600 feet high. Nine hours after starting, they are 5,350 feet high. Which equation relates their height (A) to the number of hours they have been climbing (T)?  
    Point 2: (9, 5350)  
    Point 1: (4, 4600)
11. Waylon is on an elevator in the Empire State Building that is going down. After ten seconds, he is at the 104th floor. After thirty seconds he is at the 92nd floor. Which equation relates the number of seconds (S) to the floor (F) that he is at?  
    Point 2: (30, 92)  
    Point 1: (10, 104)
12. Show how you arrived at your answers.
13. A tree is planted in the ground. The tree grows the same amount each ear. After two years the tree is 14 feet tall. After five years the tree is 26 feet tall. (a) Write an equation that relates the height of the tree (H) to the number of years since it was planted (Y). (b) use your equation to determine how tall the tree will be after ten years.  
    Point 2: (5, 26)  
    Point 1: (2, 14)
14. Kendrick exercises by doing pull-ups. Each week he increases the number of pull-ups he can do by the same amount. After four weeks of training, he can do 9 pull-ups. After ten weeks of training, he can do 18 pull-ups. (a) Write an equation that relates the number of pull-ups he can do (P) to the number of weeks he has been training (W). (b) How many pull-ups could he do when he started training? (c) After how many weeks will he be able to do 24 pull-ups?  
    Point 2: (10, 18)  
    Point 1: (4, 9)
15. Ava uploads to the Internet a video that goes viral. Each day the video is seen by a certain number more than the day before. On the sixth day, the video is watched 1,900 times. On the tenth day, the video is watched 3,100 times. (a) what equation relates the number of views (V) to the number of days (D) since the video was posted? (b) How many time will the video be watched on the 14th day? (c) How many times will the video be watched on the 31st day?  
    Point 2: (10, 3100)  
    Point 1: (6, 1900)
16. The population of Mathlandia is 300 million in 2007 and 380 million in 2015. The population increases by the same amount each year. (a) What equation can be used to relate the number of years since 2000 (T) to the population in millions (P)? (b) When will the population reach 730 million?  
    (population amounts in millions)  
    Point 2: (15, 380)  
    Point 1: (7, 300)  
     **(Year: 2050)**
17. At 8:03 A.M. there are 188 empty seats on the train. At 8:10 A.M. there are 160 empty seats. The number of empty seats decreases by the same amount each minute. (a) What equation can be used to relate the number of minutes after 8:00 A.M. (M) to the number of empty seats on the train (E)? (b) How many seats will be empty at 8:15? (c) At what time will there be 120 empty seats? (d) At what time will there be no empty seats left?   
      
    Point 2: (10, 160)  
    Point 1: (3, 188)